MATHEMATICAL MODEL FOR HOUSING ALLOCATION AND RESETTLEMENT OF HOMELESS POPULATION DUE TO BOKO HARAM INSURGENCY IN ASKIRA UBA, BIU, DAMBOA AND GWOZA LOCAL GOVERNMENT AREAS OF BORNO STATE, NIGERIA

ZAJES 24(S)2024
p-ISSN:2795-3890
e-ISSN: 2805-3877

Hassan Habib Ph.D
Department of Mathematics, College of Education,
Waka-Biu, Borno State, Nigeria.
+2348034463968
habibkantoma@gmail.com

In this study, a model of optimum resettlement pattern for internally displaced persons due to Boko Haram insurgency in Askira Uba, Biu, Damboa and Gwoza Local Government areas of Borno State that suffered all kinds of impacts by Boko Haram insurgency is defined and established. Interview and observation methods of data collection are used to collect data in the four Local Government areas which is interpreted into the ordinary differential equation for developing the model by considering the havoc of the insurgency on the individuals. The system of ordinary differential equations was solved using Cauchy's solution and the stability of the system was also analyzed using asymptotic analysis. The results established show the flow for optimum resettlement of internally displaced families in the study area which may be used as a scheme by the Borno State government in the distribution of resources to victims in any natural disaster. It is concluded that the possible extension of this work is to make the system of equations of the model non-autonomous and use a small time scale to drive the stability and asymptotic behaviours.

Article History
Received: Oct. 2024
Review processes
Oct - Nov 2024
Received in revised form: Nov 2024
Accepted: Dec 2024
Published online: Dec 2024

KEYWORDS
- Mathematical Model
- Homeless Population
- Asymptotic Analysis

Introduction

Mathematical model is an abstract description of a concrete system using mathematical concepts. The use of mathematical models to solve problems is a large part of the field of operational research. Eiselt, Bhadury and Burkey (2011). Mathematical modelling of course accentuates the beauty of mathematics and can help produce urgent results for handling insurgency thereby making it possible to achieve the housing allocation and resettlement plan. Rajan, Singh, Shubham and Vipin (2002).

Violent militia groups sprang up in the Northeastern part of Nigeria and formally denounced any ties with the federal government of Nigeria and its Western belief system. The militia strongly opposed Western influence and education. Referring to Western education as "Haram," which simply means taboo, the group has been consequently tagged Boko Haram, which means "Western education is taboo."

Boko Haram forcefully unseated local council administrators and ruled with their version of Sharia law. They established caliphates in more than thirteen local council areas in the northeastern part of the country, crippling economic, social and political activities. The group burnt houses, churches and mosques that oppose their belief system. Insani, Taheri, and Abdollahian (2024).

This reign of terror in the northeast region led to Nigeria's worst internal crisis in history, recording millions of deaths, destruction of properties, and the displacement of millions of people. Internal displacement on a large scale became a grave situation within Nigeria. The National Emergency Management Agency (NEMA) of Nigeria reported 2.1 million internally displaced persons (IDPs) at the beginning of 2016. These people represented more than 300,000 households in the northern part of the country making it the highest displacement incidence in Africa. Bettina and Walz (2009).

The ongoing insurgency in northern Nigeria, called "Boko Haram," and the government's often brutal attempts to suppress it, have produced a tide of refugees and internally displaced in one of the world's poorest regions. With the "fog of war," government restrictions on news agencies, and a poor communications infrastructure, it is difficult to survey needs with precision.

IRIN published that an estimated 350,000 people have been displaced since 2013. Of that number, 290,000 are internally displaced and the rest have fled to Cameroon, Chad, and Niger. The UN High Commissioner for Refugees (UNHCR) has a higher estimate of internally displaced persons, at 470,000. Hasti, Reza and Roya (2019). Local Nigerian officials are telling me that the real number is dramatically higher than these estimates; in Bauchi, according to one official, internally displaced persons number more than a million. They come, he said, from the ethnic conflict in Plateau state as well as from further north where "Boko Haram" is active. According to IRIN, there are no official camps for the internally displaced. They shelter with family and friends where they can find them. That reality also disguises how large the internally displaced numbers are likely to be. Eziyi and Dominic (2010).

The psychological impact of terrorism on the people in terms of displacement cannot be undermined coupled with the immediate effort to get them resettled. People living in areas affected by insurgency have migrated to other relatively safe areas in the region temporarily. The activities of the Boko Haram sect have increased the displacement of people in the form of internally displaced persons (IDPs) fleeing to safer havens within the nation and refugees fleeing into neighbouring nations. Those whose homes have been damaged or destroyed by the insurgent attacks have nowhere to go back to. Most internally displaced persons live and share resources with host communities. Johnson and Caulkins (2006).

The problem of protecting and assisting IDPs is not a new issue. In international law, it is the responsibility of the government concerned to provide assistance and protection for the IDPs in their country. However, many of the displaced in Nigeria are a result of the Boko Haram insurgency. Therefore, in this study, the interview and observation method of data collection will be to collect data in some sampled areas in the four selected Local Governments which are

1) Sampled areas in Askira Uba Local Government Area affected by Boko Haram insurgency are: i) klangar ii) Alagarno - Blakurke iii) Ngulde iv) Ngwahi v) Bapa vi) lasa vii) Wuba viii) Vile ix) Bagajau x) Maikadir xi) Yafa xii) Kirchina
2) Sampled areas in Biu Local Government Area affected by Boko Haram insurgency are:
   i) Buratai ii) Miring iii) Gunda iv) Gur v) Kurnari vi) Mandaragrau vii) Bam Buratai viii) Kamuya ix) Debiro
3) Sampled areas in Damboa Local Government Area affected by Boko Haram insurgency are: i) Sabon Gari ii) Bulablin iii) Wajirago iv) Alagarno v) Ajigin vi) Talala

4) Sampled areas in Gwoza Local Government Area affected by Boko Haram insurgency are:

i) Ngoshe ii) Pulka iii) Bita iv) Izge v) Yamtage vi) Kirawa vii) Agapalwa viiii) Wala ix) Habagda x) Kuranabasa

The large number of internally displaced persons in Borno State, combined with the inadequate housing and welfare planning by both the Borno State Government and NGOs to address the challenges created by insurgency, prompted me to conduct this research. Consequently, the number of homeless individuals in the selected study area was used to develop a model for housing allocation to internally displaced persons in these Local Government Areas, along with an analysis of the model's stability.

Statement of the problem

Inadequate housing and welfare planning by Borno State Government to cope with the problems created by activities of insurgency harms the estimation of the cost of resettlement for victims of insurgency. Nikolopoulos and Tzanetis (2003) considered a model for housing allocation of a homeless population due to a natural disaster. This model applies to any type of disaster, whether natural or man-made, as long as the affected population is displaced, considering the nature of the compartments involved in the model. In their model, they derived a non-linear system of ordinary differential equations and analysed the stability of the system limiting their system of model into five categories not taking into consideration the havoc of the natural disaster on the individuals. Therefore, in this research work, I will devise an optimum resettlement scheme to curtail housing needs and even distribute resources to victims due to Boko Haram insurgency in some selected Local Government areas of Borno State.

Objectives of the study

The objectives of this research are;

i. Derive a non-linear system of ordinary differential equations
ii. Analyze a model by considering housing allocation and resettlement of internally displaced persons due to Boko Haram insurgency in Askira Uba, Biu, Damboa and Gwoza Local Governments.
iii. Analyze the stability of the system.
iv. Design a model of the flow of settlement of internally displaced persons.

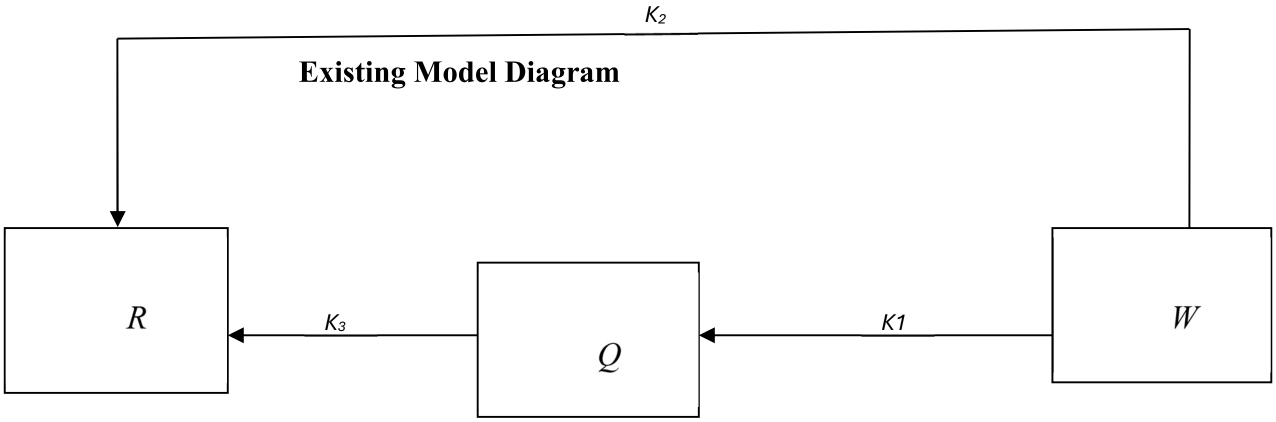
Research questions

i. What is the effect of the non-linear system of ordinary differential equations on the housing allocation and resettlement process of internally displaced persons?
ii. How can the model be analysed, considering the housing allocation and resettlement process of internally displaced persons due to Boko Haram insurgency in Askira Uba, Biu, Damboa and Gwoza Local Governments?
iii. What is the stability of the system?
iv. What is the effect of the model on the flow of settlement of internally displaced persons.

Method

Model Assumptions:

R - Number of populations resettled
Q - Number of populations accommodated in a temporary state.
W - Number of population homeless after the disaster



Existing Model Diagram

$$D'(t) = -k_1 w(Q_a - Q) - k_2 w\left(R_a - R\right) \qquad (1)$$

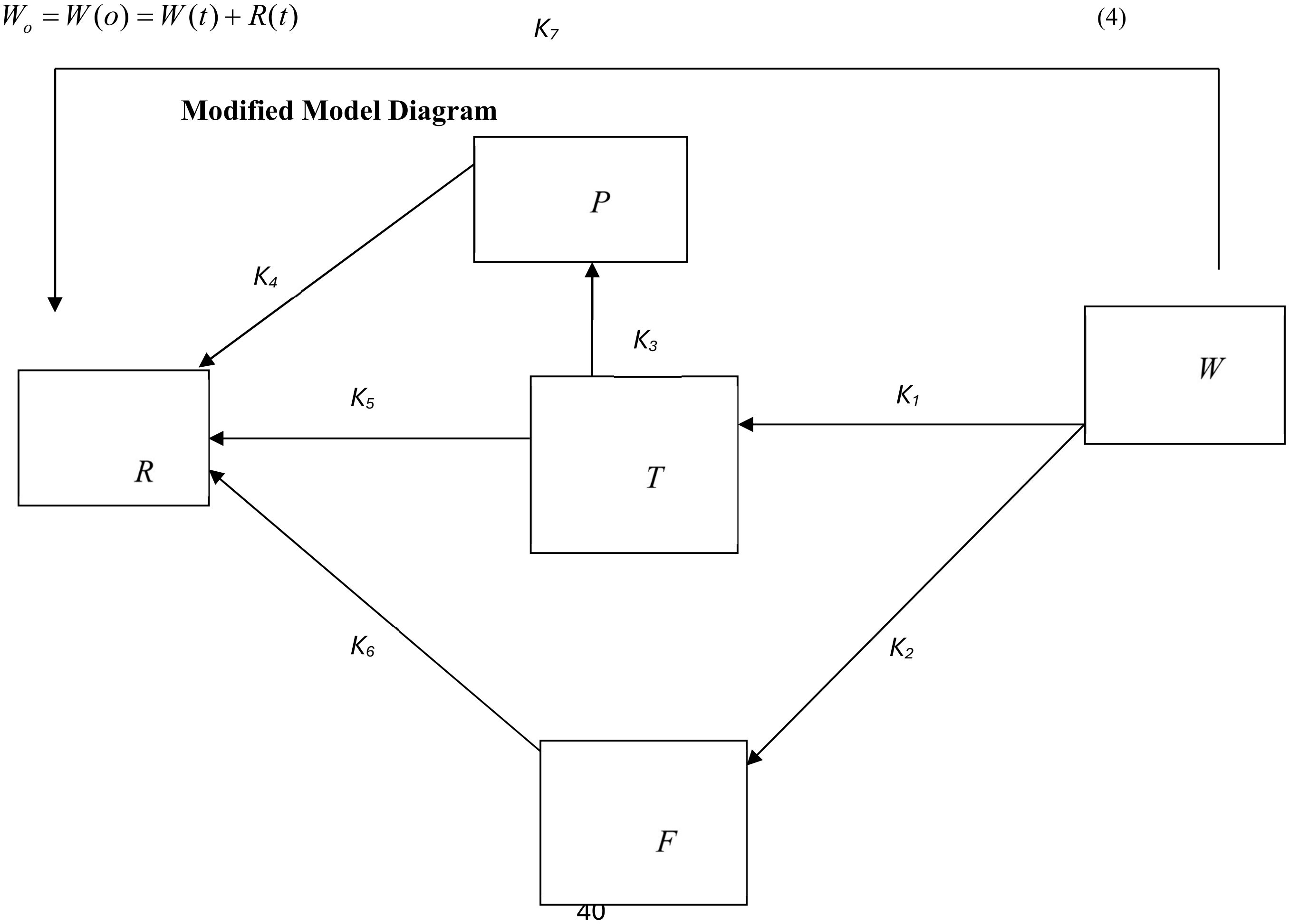
Where k_1 and k_2 are positive constants of proportionality. The rate of change in the number of populations living in temporary accommodation is proportional to $W\left(Q_a - Q\right)$ and number of populations that are resettled $Q(Ra - R)$

Thus,

$$Q'(t) - k_1 W(Q_a - Q) - k_3 Q(R_a - R) \qquad (2)$$

$$R'(t) = (k_2 w + k_3)(R_a - R) \qquad (3)$$

$$W_o = W(o) = W(t) + R(t) \qquad (4)$$



Modified Model Diagram

Model Assumptions:

R - Number of populations resettled
Q - Number of populations accommodated in temporary state.
W - Number of populations homeless after the disaster
P - Number of populations in camps by State
T - Number of populations living in available tents by state
F - Number of populations living with friends or relatives.

$$W'(t) = -k_1W(T_a - T) - k_2W(F_a - F) - k_7W(R_a - R) \tag{5}$$

$$P'(t) = -k_4P(R_a - R) + k_3T(P_a - P) \tag{6}$$

$$T'(t) = k_1W(T_a - T) - k_3T(P_a - P) - k_5T(R_a - R) \tag{7}$$

$$F'(t) = k_2W(F_a - F) - k_6F(R_a - R) \tag{8}$$

$$R'(t) = k_7W(R_a - R) + k_4P(R_a - R) + k_5T(R_a - R) + k_6F(R_a - R) \tag{9}$$

Equations 5, 6, 7, 8 and 9 are the equations ordinary differential equations obtained from the existing model diagram above by considering the number of populations resettled, accommodated in temporary state, homeless after the disaster, in camps, living in available tents by state and those living with friends or relatives.

Model Assumptions:

R - Number of individuals resettled
W - Number of individuals homeless after the insurgency
P - Number of individuals in camps by Borno State
T - Number of individuals living in available tents
F - Number of individuals living with friends or relatives.
H - Number of individuals accommodated in hospitals due to injuries sustained.

$$W'(t) = -k1W(Ta - T) - k2W(Fa - F) - k3W(Ha - H) - k10W(Ra - R) - k11(Pa - P) \tag{10}$$

$$H'(t) = k3W(Ha - H) - k8H(Ra - R) - k9H(Fa - F) \tag{11}$$

$$T'(t) = k1W(Ta - T) - k4T(Pa - P) - k6T(Ra - R) \tag{12}$$

$$F'(t) = k2W(Fa - F) - k7F(Ra - R) + k9H(Fa - F) \tag{13}$$

$$P'(t) = k4T(Pa - P) - k5P(Ra - R) + k11W(Pa - P) \tag{14}$$

$$R'(t) = k10W(Ra - R) + k5P(Ra - R) + k6T(Ra - R) + k7F(Ra - R) + k8H(Ra - R) \tag{15}$$

Equations 10, 11, 12, 13, 14 and 15 are the ordinary differential equations obtained from the modified model diagram above, derived by considering the number of individuals resettled, homeless after the insurgency, in camps, living in available tents, with friends or relatives and those accommodated in hospitals due to injuries sustained.

Results

Derivation of Non-linear System of Ordinary Differential Equations

Estimation of the $k's$ value in the modified model diagram. Given the system of equations 10, 11, 12, 13, 14 and 15. I assumed that the values R, W, P, T, F, M are consistent to the data $R_s, W_s, P_s, T_s, F_s, M_s$ from the Boko Haram insurgency in four sampled local government areas of Borno State at time $t = 3$ months. Therefore, $t = t_s$ we must have $R(t_s) = R_s = 1,550$ families, $W(t_s) = W_s = 1,022$ families, $P(t_s) = P_s = 1,225$ families, $T(t_s) = T_s = 1,071$ families, $F_1(t_s) = F_{1,s} = 2028$ families, $F_2(t_s) = F_{2,s} = 1104$ families, $F_3(t_s) = F_{3,s} = 71$ families, $F_4(t_s) = F_{4,s} = 469$ families, $M(t_s) = M_s = 710$ families.

The results of the Boko Haram insurgency are demonstrated in Table 1. In the analysis, I considered all categories and we also considered that at time $t = 0$, $W(0) = W_0 = 9,250$. We also considered as well a function. Consider the functions $H(k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}) = R(t_s) - R_s, W(t_s) - W_s, P(t_s) - P_s, T(t_s) - T_s, F(t_s) - F_s\, M(t_s) - M_s$. In the derivation of the model, I used the following assumptions.

a) Birth or death rates can be neglected.
b) The number of families is large enough so all the symbols are considered to be a non-negative real function of time.
c) There is an infinitely fast rate of rehousing.
d) I will apply the conservation principle to the number of people in the sample.

Table: 1: Summary of the data collected from the sampled areas of the study

Sampled local Government	Number of populations	Total
Askira Uba	12 villages × 250 people	3,000
Biu	9 villages × 250 people	2,250
Damboa	6 villages × 250 people	1,500
Gwoza	10 villages × 250 people	2,500

F

igure 1: Chart Description of the Sampled Areas of the Study

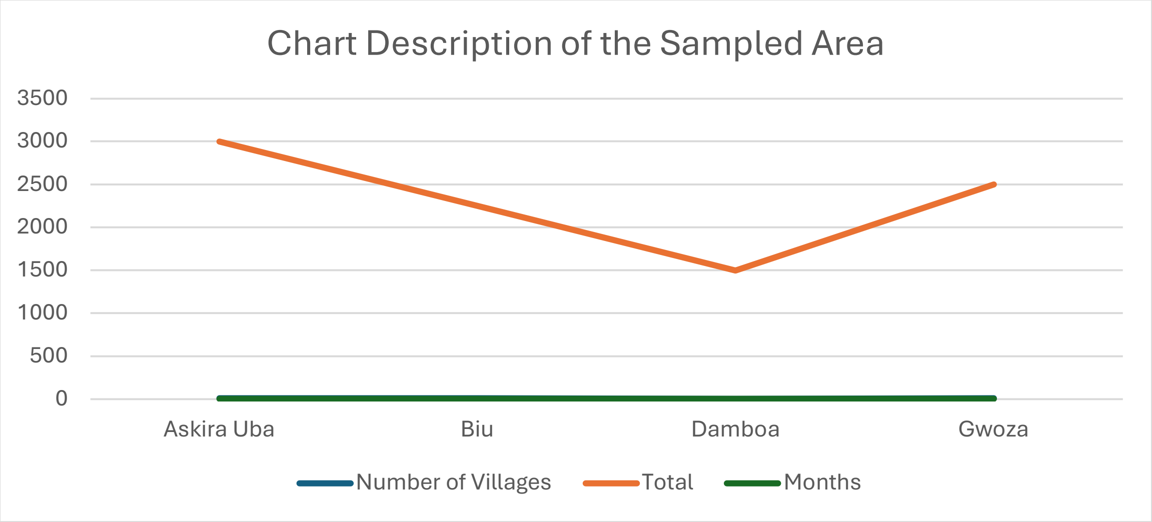


Figure 4.1 above is a chart depicting the sampled local government areas. The chart clearly shows that Askira Uba has the highest number of internally displaced persons, followed by Gwoza, Biu, and then Damboa. The blue line represents the number of villages in each local government area, the red line indicates the number of villages sampled, and the green line shows the time frame for the research.

Table 2: Sampled population from the four selected local government areas of Borno State affected by the Boko Haram insurgency

Symbols	Categories (The four sampled Local Governments)	Number of Families
R	Number of populations resettled	1,550
W	number of populations are accommodated in a temporary state.	1,022
P	Number of populations homeless after the disaster	1,225
T	Number of populations in camps by State	1,071
F_1	Guests with friends or relatives	2,028
F_2	Families living in temporary accommodation	1,104
F_3	Families renting houses	71
F_4	Families living in Schools	469
M	Number of individuals accommodated in hospitals due to injuries sustained	710
W_0	Summary of all categories	9,250

Figure 2: Chart Description of the Data Collected

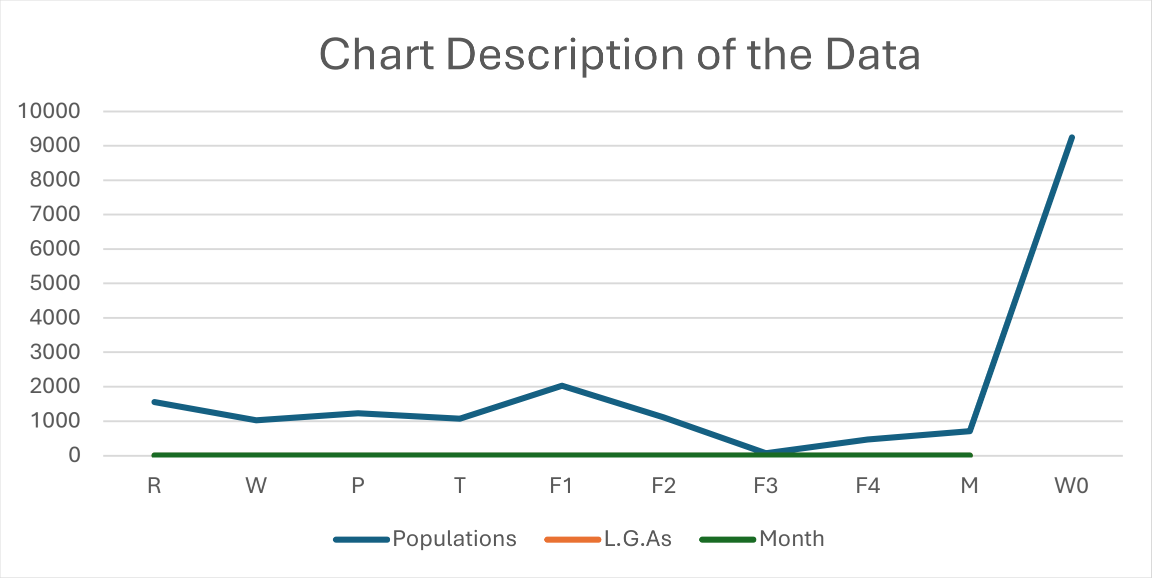


Figure 4.2 above is a chart illustrating the data collected for this research, based on the nine categories of internally displaced persons considered in the study. The blue line represents the population in each category, the red line indicates the sampled local government areas and the green line shows the time frame for the research. Solving the system of equations in the existing model numerically, as a transcendental equation with an iteration scheme. I started with an initial guess of the $k's$ value and at each step of the iteration I solved the system of the equations numerically using $RK4$ the method to give $R_s, W_s, P_s, T_s, F_s, M_s$. I make the initial guest in such a way that the values of the $k's$ increases from smallest to highest. This is because I expect to see a very quick decrease in the number of homeless populations settled in temporary accommodation supplied by Borno State government or permanent accommodation for those families that have the financial ability to do so. I also expect to have flow which is weak from people already from temporary accommodations to a permanent residence. Moreover, I also assumed that there is enough availability in both temporary and permanent accommodations that is $R_x = 100\times10^3$ units of permanent accommodation, $T_x = 80\times10^3$ units of temporary accommodation. This set of values corresponds to a situation where initially the number of homeless people decays rapidly while, because constants of availability are not very large, the majority of families are accumulated to temporary accommodation.

Note that the first of the above considerations corresponds to a situation where a respectable number of families move to permanent accommodation immediately while the second consideration corresponds to a situation where initially, families can only settle to a temporary accommodation due to financial inability. In general, I consider these two cases which be close to reality. Therefore, I consider following a system of ordinary differential equations by reducing the modified model equations.

Consider the following system of ordinary differential equations below

$$W'(t) = -k1W(Ta-T) - k2W(Fa-F) - k3W(Ha-H) - k10W(Ra-R) - k11(Pa-P) \quad (16)$$

$$H'(t) = k3W(Ha-H) - k8H(Ra-R) - k9H(Fa-F) \quad (17)$$

$$T'(t) = k1W(Ta - T) - k4T(Pa - P) - k6T(Ra - R)$$

(18)

$$F'(t) = k2W(Fa - F) - k7F(Ra - R) + k9H(Fa - F)$$

(19)

$$P'(t) = k4T(Pa - P) - k5P(Ra - R) + k11W(Pa - P)$$

(20)

$$R'(t) = k10W(Ra - R) + k5P(Ra - R) + k6T(Ra - R) + k7F(Ra - R) + k8H(Ra - R)$$

(21)

I assumed throughout this study that the population would remain constant during this research process and therefore

$$W_0 = W(0) = W(t) + H(t) + T(t) + F(t) + P(t) + R(t)$$

(22)

Finally, using equation (22)

substitute equation (16) in (17) we have

$$\begin{aligned} x'(t) &= W'(t)H'(t) \\ &= -k_1k_{11}w^2(T_a - T)(P_a - P) - k_2k_9wH(F_a - F) - k_3k_8k_{10}w^2H(H_a - H)(R_a - R) \\ &= -x_1z - x_2a - x_3y \end{aligned}$$

(23)

where $x_1 = k_1k_{11}w^2$, $z = (T_a - T)(P_a - P)$

$x_2 = k_2k_9wH$ $a = (F_a - F)$ $x_3 = k_3k_8k_{10}w^2H$

$y = (H_a - H)(R_a - R)$

substitute equation (18) in (19) we have

$$\begin{aligned} y'(t) &= T'(t)F'(t) \\ &= k_1k_2w(T_a - T)(F_a - F) - k_4k_6k_7k_9T^2HF(P_a - P)(R_a - R) \\ &= x_1y_1 - by_2 \end{aligned}$$

(24)

where $x_1 = k_1k_2w$, $y_1 = (T_a - T)(F_a - F)$,

$b = k_4k_6k_7k_9T^2HF$, $y_2 = (P_a - P)(R_a - R)$

substitute equation (20) in (21) we have

$$\begin{aligned} z'(t) &= P'(t)R'(t) \\ &= k_4k_{11}Tw(P_a - P) - k_5k_6k_7k_8k_{10}P^2wTF(R_a - R) \\ &= x_1z_1 - cz_2 \end{aligned}$$

(25)

where $x_1 = k_4k_{11}Tw$, $z_1 = (P_a - P)$,

$c = k_5k_6k_7k_8k_{10}P^2wTF$, $z_2 = (R_a - R)$

to have equations (26), (27) and (28) respectively with

$$x'(t) = -xz - ax - xy$$

(26)

$$y'(t) = xy - by$$

(27)

$$z'(t) = xz - cz$$

(28)

where $x(t) \geq 0$, $y(t) \geq 0$, $z(t) \geq 0$ with $a, b, c > 0$

Stability of the Stationary Points $(0,0,0)$, $(b,a,0)$ and $(c,0,a)$

To investigate the stability of the system, consider the equilibrium points i. e stationary, steady-state, fixed point which are solutions of equations (26), (27) and (28). The stability of this point also depends crucially on the value of the eigenvalues λ . i.e. on the availability for resettlement. If λ is positive, then the system is asymptotically stable and if λ is negative then the point is unstable, and if $\lambda = 1$ then further analysis will be required to investigate the possible points.

Where λ eigenvalue is obtained from the system of ordinary differential equations (26), (27) and (28) which is used to interpret the stability of the system.

Analysis of the Model

Center Manifold Analysis

The following theorem characterizes all equilibrium points. Equating (26), (27) and (28) to zero solving we obtain the following equilibrium point because I expect to have no homeless families in temporary accommodation after enough time has elapsed.

Equation (29) is obtained by equating the differential equation (26) to zero to obtain the equilibrium point

$$-\ xz - ax\ - xy = 0$$

(29)

Equation (30) is obtained by equating the differential equation (27) to zero to obtain the equilibrium point

$$xy\ - by = 0$$

(30)

Equation (31) is obtained by equating the differential equation (28) to zero to obtain the equilibrium point

$$xz\ -\ cz = 0$$

(31)

solving equation (29) gives

$ax\ - xy\ -\ xz = 0$, when $x = 0$, we have $(0,0,0)$

solving equations (30) gives

$-by + xy = 0$, $y(-b + x) = 0$, $y = 0$ or $-b + x = 0 \Rightarrow x = b$ and

solving equations (31) gives

$ax - xy = 0$, $x(a - y) = 0$, $x = 0$ or $y = a$ we have $(b,a,0)$

$-\ cz + xz = 0$, $z(-\ c + x) = 0$, $z = 0$ or $x = c$ $ax\ -\ xz = 0$, $x(a\ -\ z) = 0$, $x = 0$ or $z = a$ we have $(c,0,a)$

Based on the solutions of equations (29), (30) and (31), the three equilibrium points obtained are as follows

Equilibrium points $(0,0,0)$ obtained from the solution of equation (29)

Equilibrium points $(b,a,0)$ obtained from the solution of equation (30) and

Equilibrium points $(c,0,a)$ obtained from the solution of equation (30)

Theorem 4. 1: System (29), (30) and (31) have three equilibrium points:

- $(0,0,0)$ which is a saddle point,
- $(b,a,0)$ which is a nonhyperbolic point, and
- $(c,0,a)$ which is a nonhyperbolic point,
  where and a,b,c are positive

Proof: Consider the continuous map obtained by combining equations (29), (30) and (31)

$$f(x) = \begin{pmatrix} ax\ -\ xy\ -\ xz \\ -by\ +\ xy \\ -cz\ +\ xz \end{pmatrix}$$

(32)

Clearly, $f(x) = 0$ at

$x_1 = (0,0,0)^T$,

$x_2 = (b,a,0)^T$ and

$x_3 = (c,0,a)^T$.

We first compute the Jacobian matrix of partial derivatives:

$$Df = \begin{pmatrix} a - y - z & -x & -x \\ y & -b + x & 0 \\ z & 0 & -c + x \end{pmatrix}$$

(33)

The derivative at equilibrium point x_1

$$Df(x_1) = \begin{pmatrix} a & 0 & 0 \\ y & -b & 0 \\ z & 0 & -c \end{pmatrix}$$

(34)

To find the eigenvalues,

$$Df(x_1) = \begin{pmatrix} a & 0 & 0 \\ y & -b & 0 \\ z & 0 & -c \end{pmatrix} = \begin{vmatrix} a - \lambda & 0 & 0 \\ y & -b - \lambda & 0 \\ z & 0 & -c - \lambda \end{vmatrix}$$

$$a - \lambda \begin{vmatrix} -b - \lambda & 0 \\ 0 & -c - \lambda \end{vmatrix},$$

$$a - \lambda\ \big((-b - \lambda)(-c - \lambda)\big) = 0$$

$$(a - \lambda)\big(\lambda^2 + b\lambda + c\lambda + bc\big) = 0$$

$$\lambda_1 = a,\ \lambda_2 = b,\ \lambda_3 = c$$

$Df(x_1)$ has eigenvalues with different signs, from which we conclude that x_1 is a saddle point So, equilibrium point x_1 is unstable.

The derivative at equilibrium point x_2

$$Df(x_2) = \begin{pmatrix} 0 & -b & -b \\ a & 0 & 0 \\ z & 0 & -c + b \end{pmatrix}$$

(35)

To find the eigenvalues,

$$Df(x_2) = \begin{pmatrix} 0 & -b & -b \\ a & 0 & 0 \\ z & 0 & -c+b \end{pmatrix} = \begin{vmatrix} -\lambda & -b & -b \\ a & -\lambda & 0 \\ 0 & 0 & -c+b-\lambda \end{vmatrix}$$

$$-\lambda \begin{vmatrix} -\lambda & 0 \\ 0 & -c+b-\lambda \end{vmatrix} + b \begin{vmatrix} a & 0 \\ 0 & -c+b-\lambda \end{vmatrix} = 0$$

$$-\lambda(\lambda c - \lambda b + \lambda^2) + b(-ac + ab - a\lambda) = 0$$

$$-\lambda^3 + \lambda^2 b - \lambda^2 c + ab^2 - abc - ab\lambda = 0$$

$$\lambda^3 - \lambda^2(b-c) + ab\lambda - ab(b-c) = 0$$

$b-c$ is one of the factors of the above equation, therefore, $\lambda_1 = b-c$

Applying long division to the above polynomial, we have the following

$$\begin{array}{r} -\lambda^2 + a\lambda - ab - a\lambda \\ b-c\,\overline{)\,-\lambda^3 + \lambda^2 b - \lambda^2 c + ab^2 - abc - ab\lambda} \end{array}$$

$$-$$

$$-\lambda^2 b + \lambda^2 c$$

$$ab\lambda - ab^2 + abc$$

$$-$$

$$ab\lambda - ac\lambda$$

$$-ab^2 - ac\lambda + abc$$

$$-$$

$$-ab^2 + abc$$

$$-ac\lambda$$

$$-$$

$$-ac\lambda$$

$$0$$

$$-\lambda^2 + a\lambda - ab - a\lambda = 0$$

$$-\lambda^2 - ab = 0$$

$$\lambda^2 = -ab$$

$$\lambda = \pm i\sqrt{ab}$$

And corresponding eigenvalues are $\lambda_1 = b - c$, $\lambda_2 = i\sqrt{ab}$ and $\lambda_3 = -i\sqrt{ab}$. Since eigenvalues λ_2 and λ_3 are pure imaginary, then equilibrium x_2 is a non-hyperbolic point. So, we cannot conclude anything at this stage about the stability of this equilibrium point. Similarly,

$$Df(x_3) = \begin{pmatrix} 0 & -c & -c \\ 0 & -b+c & 0 \\ a & 0 & 0 \end{pmatrix} \quad (36)$$

To find the eigenvalues,

$$Df(x_3) = \begin{pmatrix} 0 & -c & -c \\ 0 & -b+c & 0 \\ a & 0 & 0 \end{pmatrix} = \begin{vmatrix} 0 & -c & -c \\ 0 & -b+c & 0 \\ a & 0 & 0 \end{vmatrix}$$

$$-\lambda \begin{vmatrix} -b+c-\lambda & 0 \\ 0 & -\lambda \end{vmatrix} - c \begin{vmatrix} 0 & -c+b-\lambda \\ a & 0 \end{vmatrix} = 0$$

$$-\lambda(-\lambda c + \lambda b + \lambda^2) - c(-ac + ab + a\lambda) = 0$$

$$\lambda^3 + \lambda^2 b - \lambda^2 c - ac^2 + abc + ac\lambda = 0$$

$$\lambda^3 - \lambda^2(c-b) + ac\lambda - ac(c-b) = 0$$

$c-b$ is one of the factors of the above equation, therefore, $\lambda_1 = c-b$

Applying long division to the above polynomial, we have the following

$$\begin{array}{r} -\lambda^2 + a\lambda - ac - a\lambda \\ c-b\,\overline{)\,\lambda^3 + \lambda^2 b - \lambda^2 c - ac^2 + abc + ac\lambda} \end{array}$$

$$-$$

$$-\lambda^2 c + \lambda^2 b$$

$$ac\lambda - ac^2 + abc$$

$$-$$

$$ac\lambda - ab\lambda$$

$$-ac^2 - ab\lambda + abc$$

$$-$$

$$-ac^2 + abc$$

$$-ab\lambda$$

$$-$$

$$-ab\lambda$$

$$0$$

$$-\lambda^2 + a\lambda - ac - a\lambda = 0$$

$$-\lambda^2 - ac = 0$$

$$\lambda^2 = -ac$$

$$\lambda = \pm i\sqrt{ac}$$

And $\lambda_1 = c - b$, $\lambda_2 = i\sqrt{ac}$ and $\lambda_3 = -i\sqrt{ac}$, from which we conclude that x_3 is a non-hyperbolic point and we cannot conclude anything about the stability of this equilibrium point.

Stability Analysis of the System

Asymptotic Analysis

In the next theorem, we will predict the behaviour of the resettlement model and the resettlement of the internal displacement by considering the theorem below

$$x'(t) = -by \quad (37)$$

$$y'(t) = -cz \quad (38)$$

and analyzing x axis we can predict the resettlement of the internal displacement.

Model of Flow of Settlement of Internal Displaced Persons

Derivation of a More Sophisticated Model

Consider the following system of ordinary differential equations below

$$x'(t) = ax - bxy \quad (39)$$

$$y'(t) = cxy - dy \quad (40)$$

With eigenvalues $\lambda_1 = c-b$, $\lambda_2 = i\sqrt{ac}$ and $\lambda_3 = -i\sqrt{ac}$. Using the theory of center manifold analysis. The eigenvalue corresponds to $\lambda_1 = c-b$ $\lambda_2 = i\sqrt{ac}$ and $\lambda_3 = -i\sqrt{ac}$ is predicted as the existence of curves invariant under the flow and tangent, where the function $x(t)$ represents the total population of the study area at time $t = 3$ Months and also the function $y(t)$ represents the populations in the temporary tent by the Borno State government also at time $t = 3$ Months with a, b, c, d positive constants.

The modified resettlement model is given by the following system

$$x'(t) = ax - bx^2 - cxy \quad (41)$$

$$y'(t) = -dy + exy - fy^2 \quad (42)$$

Where the scale quantities representing the number of families in temporary accommodations.

The modified three-dimensional resettlement model also uses a nonlinear system of equations

$$x'(t) = ax - bx^2 - cxy \quad (43)$$

$$y'(t) = fxy - ey \quad (44)$$

$$z'(t) = ixz + jyz - gz - hzx \quad (45)$$

Using a change of coordinates, system (43) can be transformed into the system of ordinary differential equation (46)

$$x'(t) = x - bx^2 - xy - xz \quad (46)$$

Using a change of coordinates, system (44) can be transformed into the system of ordinary differential equations (47)

$$y'(t) = xy - by \quad (47)$$

Using a change of coordinates, system (45) can be transformed into the system of ordinary differential equations (48)

$$z'(t) = exz + fyz - cz - dzx \quad (48)$$

The transformations in equations (43), (44) and (45) above which gives equations (46), (47) and (48) will be unstable for positive initial conditions and the equilibrium point $(0,0,0)$ which is the origin, is an attractor.

Since equations (46), (47) and (48) are unstable for positive initial conditions and the equilibrium point $(0,0,0)$ is an attractor, this system of ordinary differential equations is true because I expect to have no homeless families or families in the temporary accommodation after enough time has elapsed.

Discussion

Two models were derived regarding the movement of the homeless population due to Boko Haram insurgency in four selected local government areas of Borno State. In the second of these, the number of families in temporary accommodation was divided into three categories while in the first model, it was taken as one category. In both cases, I analyzed the stability of the system of derived ordinary differential equations. I obtained the steady states of the system and looked at the zero solutions because I am interested in the state of the system where the origin is attracting with the presence of zero eigenvalues when the availability of the accommodation is equal to the demand. The Jacobian matrix of the system for this steady state indicates that more analysis is needed to be able to see the behaviour of the system with positive initial conditions. Also, an asymptotic solution of the system was given for the first model based on its determinant flow. The models were solved and the results were analyzed.

All the numerical estimates are based on the data taken from the four sampled local government areas of Borno State which are Askira Uba, Biu, Danmboa and Gwoza local government areas.

Conclusion

In this research work, I derived a model by considering the number of homeless population due to Boko Haram insurgency. I analyzed the flow of this population to a temporary state (tends, prefabricated houses, relatives etc.) until their resettlement in a permanent residence.

A fairly simple model was derived. This model consists of a system of ordinary differential equations. The population was divided into nine (9) categories regarding their allocation state. These are a) the number of populations resettled b) the number of populations accommodated in a temporary state c) Number of populations homeless after the disaster d) Populations in camps f) Guests in friends or relatives g) Families living in temporary accommodation h) Families renting houses i) Families living in Schools j) Individuals accommodated in hospitals due to injuries sustained. Modelling the flow between them leads to a system of ordinary differential equations.

I also studied the possible steady state of the system and analyzed the stability of the model for the possible positive equilibrium points. For some specific cases, the stability of the system cannot be deduced directly and central manifold analysis was applied.

Moreover, data from sampled study areas which are Askira Uba, Biu, Damboa and Gwoza local government areas of Borno State were considered in the estimation of the flows in the model. Also, for certain choices of the coefficient of the flow, the asymptotic solution of the system was obtained.

A similar analysis to a more sophisticated version of this model was applied by considering the number of families in temporary accommodations divided among those living in camps organized by Borno State government, those living in self-provided temporary accommodation and those living with relatives and friends' houses.

Prediction of at least qualitative characteristics of homeless and non-homeless population flows could lead to better programming by the Borno State government to cope with the problems created by the Boko Haram insurgency. Planning appropriate stock of tents or prefabricated houses etc. also another aspect could be the estimation of the cost and time of resettlement for the victims of the Boko Haram insurgency. The results obtained will be useful to the welfare planning of Borno State government for housing allocations and distribution of resources to IDPs etc.

Recommendations

The model obtained in this research work was based on the assumption that the availability of accommodation is constant. This means that initially there was a sufficient number of tents or accommodations in an organized camp and fabricated houses supplied by the Borno State government according to the demand. Another approach will be to have $R_s, W_s, P_s, T_s, F_s, M_s$ as function of time and assume for example that $R_s = W_s = P_s = T_s = F_s = M_s$. This would express the fact that fabricated houses are supplied after some time. This modification will make the system of equations of the model non-autonomous which will be considered in such a case that the time scale of the system is small and drives the stability and asymptotic behaviours.

Another modification would be to consider the time that the populations or families need to settle in temporary accommodation or to resettle as far as such a possibility is available for the families. This would lead to a system of delayed ordinary differential equations.

Contribution to knowledge

i. Mathematical model for optimum resettlement of internally displaced families due to Boko haram insurgency is established.

ii. New system of ordinary differential equations based on the established mathematical model is formulated and solved.

iii. New mathematical scheme is formulated based on the data collected in the study area which can be used Borno State government, NGOs, Researchers etc. for resettling the IDPs as well as distribution of relief materials.

Acknowledgement

The research work was supported by the Tertiary Education Trust Fund (TETFUND), Federal Republic of Nigeria. It was financially supported by the IBR TETFund Research Project (Batch-4 2024)

References

Bettina, K. & Walz, H. (2009). *Farsighted House Allocation*. Departement de Sciences Economiques and CIREQ. Universite de Montreal, Montreal, Quebec H3C 3J7, Canada.

Eziyi, j. & Dominic, G. (2010). *Evaluation of the impact of public housing programmes on sustainable development in Nigeria.*

Eiselt, H. A., Bhadury, J. & Burkey, M. L. (2011). An Optimization-Based Framework for Modeling Counterterrorism Strategies. Online at https://mpra.ub.uni-muenchen.de/47260/MPRA Paper No. 47260, posted 29 May 2013 13:20 UTC.

Hasti, S., Reza T. M. & Roya, S. (2019). A two-stage mathematical model for evacuation planning and relief logistics in a response phase. Journal of Industrial and Systems Engineering Vol. 12, No. 1, pp. 129-146.

Insani, N., Taheri, S. and Abdollahian, M. A. (2024). A Mathematical Model for Integrated Disaster Relief Operations in Early-Stage Flood Scenarios. Mathematics 2024, 12, 1978. https://doi.org/10.3390/ math12131978.

Johnson, M. P. & Caulkins, J. P. (2006). Can Housing Mobility Programs Make a Long-Term Impact on the Lives of Poor Families and the Health of Middle-Class Communities: A Policy Simulation. Heinz School Working Paper Series 2005-33; under review at *Housing Policy Debate*.

Lippman, B. J. (2006). National Housing Conference. *A Heavy Load: The Combined Housing and Transportation Burdens of Working Families*. Washington, D.C.: Center for Housing Policy.

http://www.nhc.org/pdf/pub_heavy_load_10_06.pdf.

Nikolopoulos, C.V. & Tzanetis, D. E. (2003). A model for housing allocation of a homeless population due to natural disasters. *Public Policy and Public Affairs Faculty Publication Series.* Paper 4. http://scholarworks.umb.edu/pppa_faculty_pubs/4.

Rajan, S., Singh, B. K., Shubham, K. & Vipin, K. (2002). Mathematical Modeling of Effective Distribution System for Resources and Facilities in Disaster Management by using Optimization Techniques. Mathematical Statistician and Engineering Applications ISSN:2094-0343,2326-9865. Vol 71 No. 3 (2022).