ZAJES
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ZARIA JOURNAL OF EDUCATIONAL STUDIES 24(S)2024

ZAJES 24(S)2024
p-ISSN:2795-3890
e-ISSN: 2805-3877

MATHEMATICAL MODEL OF THE DYNAMICS OF CORRUPTION CONSIDERING LOSING IMMUNITY OF EX - CONVICT

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In this study, a mathematical model was developed to describe the dynamic of corruption using epidemiolocal modelling approach, considering loss immunity of Ex – Convict. The population is divided into five compartments consisting of susceptible S(t), Exposed E(t), Corrupt C(t), Jailed J(t) and Reformed R(t). The model's equilibria are identified, and the stability of these equilibria is studied in depth. At the corruption free equilibrium point (CFEP), the next generation matrix technique is used to estimate the corruption reproduction number (R_0). The CFEP is stable when $R_0 < 1$, however, when $R_0 > 1$, then corruption would persist in society. Furthermore, the sensitivity of the model parameters was investigated, and recommendations were made. Lastly, a numerical solution was performed to confirm the analytical solutions.

Article History
Received: Oct. 2024
Review processes
Oct - Nov 2024
Received in revised form: Nov 2024
Accepted: Dec 2024
Published online: Dec 2024

KEYWORDS
- Mathematical Model
- Corruption
- Ex – Convict
- Stability
- Sensitivity Analysis

Introduction

Corruption is a social problem that persists in most organizations (Corruption Index, 2018), particularly in the government where public servants are perceived with influence or power of some kind and are expected to provide some service to the citizens they service (Jederick, 2010). Transparency International adopted the definition by Eicher (2009): "corruption is the abuse of entrusted power for private gain". Corruption in public service results in revenue losses for government, degeneration of social justice, violation of human rights and exploitation of vulnerable people (Verma and Sengupta, 2015). It is a complex phenomenon with economic, social, political and cultural dimensions which cannot easily be eliminated (Hathroubi, 2017). Due to increase of public interest and concern over the universal threat to humanity, several studies have proposed mathematical models to understand and analyze the dynamics of corruption (Abdulrahman, 2014).

The epidemiological dynamics of corruption transmission model was developed and analyzed by many authors, including: Abdulrahman (2014), Legesse and Shiferaw (2018), Felix *et al.* (2017), Zerihum and Abyneh (2022), Abayneh and Zerihun (2022), Aloke (2023), Gutema *et al.* (2024), Alhassan *et al.* (2024). All the models have served the purpose for which it was developed, however, losing Immunity of ex-convict after reform was not considered.

In this paper, reunion with Ex – Convict which occur as a result of loss of immunity is considered. The objective of this work is to describe the transmission process of corruption, which can be defined generally as follows: when a reasonable number of corrupted individuals are introduced into a susceptible population, the corruption is passed to other individuals through its modes of transmission, thus, spreading in the population. Thus, in this article, the model for the spread of corruption in the sprite of epidemiology is presented which has described the dynamical behavior of corruption, as a disease incorporating losing immunity of ex – convict.

Model formulation

The total population is divided into five non – intersecting compartments of susceptible $S(t)$, Exposed $E(t)$, Corrupt $C(t)$, Jailed $J(t)$ and Reformed $R(t)$. We assume that, susceptible individuals are innocent individuals who are not engaged in any corrupt activities, Exposed individuals are those who are exposed to a corrupted person but do not perform it, corrupt individuals are those who are engaged in corrupt activities and are capable of influencing others to become corrupt, jailed individuals comprise of those who are convicted and sentenced for corruption – related offenses, while those who undergo rehabilitation in jail, comprehending corruption's impact, are considered as reformed individuals.

Table 1: Variables and their description

Variables	Description
S	Population of susceptible individuals
E	Population of exposed individuals
C	Population of corrupted individuals
J	Population of jailed individuals
R	Population of reformed individuals

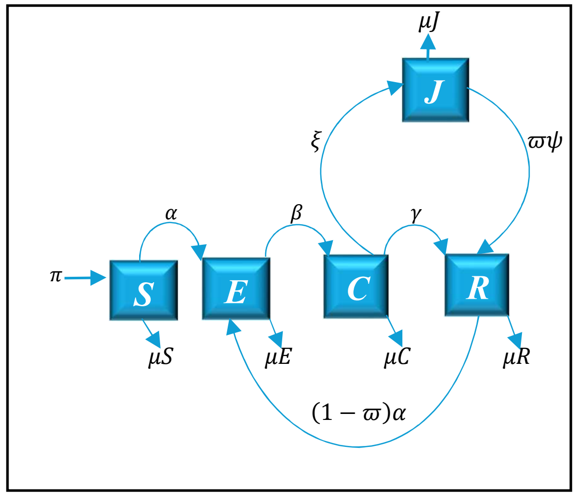

Figure 1: schematic diagram of the model

$$\left.\begin{aligned}
\frac{dS}{dt} &= \pi - \alpha SC - \mu S \\
\frac{dE}{dt} &= \alpha SC + (1-\varpi)\alpha CR - (\beta+\mu)E \\
\frac{dC}{dt} &= \beta E - (\xi + \gamma + \mu)C \\
\frac{dJ}{dt} &= \xi C - (\psi\varpi + \mu)J \\
\frac{dR}{dt} &= \gamma C + \psi\varpi J - \big((1-\varpi)\alpha C + \mu\big)R
\end{aligned}\right\} \quad (1)$$

Subject to the initial conditions
$S(0) > 0,\ E(0) > 0,\ C(0) > 0,\ J(0) > 0,\ R(0) > 0$

Table 2: Parameters and their description

Parameter	Description
π	Recruitment rate
α	Effective corruption contact rate
μ	Death rate
β	Rate at which Exposed individual become corrupt
γ	Reformed rate of the corrupt individuals
ξ	Rate at which corrupt individual move to jail
ψ	Rate at which jailed individual are reformed
ϖ	Immunity of the reformed individual
$(1-\varpi)\alpha$	Loss of immunity by reformed individual

Model Analysis

In this section, the solution of equation (1) is studied in the epidemiologically feasible region.

Positivity of solutions

Since the model (1) monitors human population, all the variables and the associated parameters are non – negative at all time. It is important to show that the variables of the model remain non – negative for all non – negative initial conditions. This will be established by the following theorem:

Theorem 1. *The region* $\mathfrak{D} = \{(S, E, C, J, R) \in \mathbb{R}_+^5\}$ *is positively invariant and attracts all solutions in* $\mathbb{R}_+^5$.

Proof

Assume that all the state variables are continues. Then, from the system of equation (1), one can easily obtained that

$$\left.\begin{aligned} \frac{dS}{dt} &\ge -\alpha SC - \mu S \\ \frac{dE}{dt} &\ge -(\beta+\mu)E \\ \frac{dC}{dt} &\ge -(\xi+\gamma+\mu)C \\ \frac{dJ}{dt} &\ge -(\psi\varpi+\mu)J \\ \frac{dR}{dt} &\ge -((1-\varpi)\alpha C+\mu)R \end{aligned}\right\} \quad (2)$$

Solving system (2), we obtained

$$\left.\begin{aligned} S(t) &\ge S_0 e^{-\mu t} \ge 0 \\ E(t) &\ge E_0 e^{-(\beta+\mu)t} \ge 0 \\ C(t) &\ge C_0 e^{-(\xi+\gamma+\mu)t} \ge 0 \\ J(t) &\ge J_0 e^{-(\psi\varpi+\mu)t} \ge 0 \\ R(t) &\ge R_0 e^{-\mu t} \ge 0 \end{aligned}\right\} \quad (3)$$

Thus, we can conclude that all the solutions are non – negative in $\mathbb{R}_+^n$ for all $t \ge 0$.

Invariant Region

If a solution of differential equation or system of differential equations starts on a given space, surface or curve (manifold or set) and remain within it for all time, then the manifold or set is said to be invariant (Okuonghae, 2017). Hence a positively invariant set or manifold will have solution that are positive for all time.

The dynamics of the system (1) can be studied in $\mathfrak{D}$ and it can be shown that $\mathfrak{D}$ is positively invariant and attractor of the feasible solution set of the system (1).

Theorem 2: *The initial conditions of the system (1) are contained in the region* $\mathfrak{D} \in \mathbb{R}_+^5$, *defined by* $\mathfrak{D} = \left\{(S, E, C, J, R) \in \mathbb{R}_+^5 : N \le \frac{\pi}{\mu}\right\}$

Proof

The rate of change of the total human population is given as

$$\frac{dN}{dt} = \pi - \mu N \tag{4}$$

Rewrite (4) in the form $\frac{dy}{dt} + Q(x) = P(x)$, we have $\frac{dN}{dt} + \mu N = \pi$

Using integrating factor method

$$N = \frac{\pi}{\mu} + Ce^{-\mu t} \tag{5}$$

At $t = 0$, $N(0) = N_o$ which give

$$C = N_o - \frac{\pi}{\mu} \tag{6}$$

Substitute (6) in (5)

By standard comparison theorem, it can be shown that $N(t) \le \frac{\pi}{\mu}$ if $N(0) \le \frac{\pi}{\mu}$

So that $\mathfrak{D}$ is positively invariant set. Thus, all solution enters $\mathfrak{D}$ and remain non – negative for initial conditions.

Remark 1

In the region $\mathfrak{D}$, the proposed mathematical model is mathematically well posed.

Corruption Free Equilibrium Point

Corruption – Free Equilibrium (CFE) points E_0 are steady state solutions, where there is no corruption in the society. Thus, CFE of the system (1) is attained when all the variables and parameters related to corruption are zero ($E = 0$, $C = 0$, $J = 0$, $R = 0$, $\alpha = 0$). Setting the RHS of (1) to zero, we have

$$\left.\begin{aligned} &\pi - \alpha SC - \mu S = 0 \\ &\alpha SC + (1-\varpi)\alpha CR - (\beta + \mu)E = 0 \\ &\beta E - (\xi + \gamma + \mu)C = 0 \\ &\xi C - (\psi\varpi + \mu)J = 0 \\ &\gamma C + \psi\varpi J - \big((1-\varpi)\alpha C + \mu\big)R = 0 \end{aligned}\right\} \tag{7}$$

Substituting $E = 0,\ C = 0,\ J = 0,\ R = 0$ in (7) and solving for S, we have

$$E_0 = \left(\frac{\pi}{\mu}, 0,0,0,0\right)$$

Basic Reproduction Number

The basic reproduction number (R_0) is define as number of newly corrupters produced by a typical corrupt individual in a complete susceptible population. The next generation matrix by Dickmann, Heesterbeek and Roberts, (2009) is applied to calculate the R_0 by considering the Exposed, Corrupted and Jailed compartments of equation (1). At the free steady state, $S = R$. This means the system has three corrupting states: E, C and J

$$\frac{dE}{dt} = \alpha SC + (1-\varpi)\alpha CR - (\beta + \mu)E$$

$$\frac{dC}{dt} = \beta E - (\xi + \gamma + \mu)C$$

$$\frac{dJ}{dt} = \xi C - (\psi\varpi + \mu)J$$

Now, we want to linearize the corrupters system, therefore let's set

$$f_1 = \alpha SC + (1-\varpi)\alpha CR - (\beta + \mu)E$$

$$f_2 = \beta E - (\xi + \gamma + \mu)C$$

$$f_3 = \xi C - (\psi\varpi + \mu)J$$

Let matrix F represent the rate of appearance of new corrupters into the compartments and V represent the rate of transmission into (out) of compartments. Then, we have

$$F = \begin{pmatrix} \alpha SC + (1-\varpi)\alpha CR \\ 0 \\ 0 \end{pmatrix}$$

$$V = \begin{pmatrix} (\beta+\mu)E \\ -\beta E + (\xi+\gamma+\mu)C \\ -\xi C + (\psi\varpi+\mu)J \end{pmatrix}$$

$$F = \begin{pmatrix} \frac{\partial f_1}{\partial E} & \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial J} \\ \frac{\partial f_2}{\partial E} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial J} \\ \frac{\partial f_3}{\partial E} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial J} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{\alpha\pi}{\mu} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (8)$$

$$V = \begin{pmatrix} \frac{\partial f_1}{\partial E} & \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial J} \\ \frac{\partial f_2}{\partial E} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial J} \\ \frac{\partial f_3}{\partial E} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial J} \end{pmatrix}$$

$$= \begin{pmatrix} \beta+\mu & 0 & 0 \\ -\alpha & \gamma+\xi+\mu & 0 \\ 0 & -\xi & \psi\varpi+\mu \end{pmatrix} \quad (9)$$

Therefore, the inverse of V is given by

$$V^{-1} = \begin{pmatrix} \frac{1}{\beta+\mu} & 0 & 0 \\ \frac{\beta}{(\beta+\mu)(\gamma+\xi+\mu)} & \frac{1}{(\psi\varpi+\mu)(\gamma+\xi+\mu)} & 0 \\ \frac{\beta\xi}{(\psi\varpi+\mu)(\beta+\mu)(\gamma+\xi+\mu)} & \frac{\xi}{(\psi\varpi+\mu)(\gamma+\xi+\mu)} & \frac{1}{(\psi\varpi+\mu)} \end{pmatrix} \quad (10)$$

$$FV^{-1} = \begin{pmatrix} 0 & \frac{\alpha\pi}{\mu} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\beta+\mu} & 0 & 0 \\ \frac{\beta}{(\beta+\mu)(\gamma+\xi+\mu)} & \frac{1}{(\psi\varpi+\mu)(\gamma+\xi+\mu)} & 0 \\ \frac{\beta\xi}{(\psi\varpi+\mu)(\beta+\mu)(\gamma+\xi+\mu)} & \frac{\xi}{(\psi\varpi+\mu)(\gamma+\xi+\mu)} & \frac{1}{(\psi\varpi+\mu)} \end{pmatrix} \quad (11)$$

$$FV^{-1} = \begin{pmatrix} \frac{\alpha\beta\pi}{\mu(\beta+\mu)(\gamma+\xi+\mu)} & \frac{\alpha\pi}{(\psi\varpi+\mu)(\gamma+\xi+\mu)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (12)$$

The dominant eigenvalue of (12) is equal to R_0, therefore, we evaluate the characteristic equation $|FV^{-1} - \lambda I| = 0$

$$\left| \begin{pmatrix} 0 & \frac{\alpha\pi}{\mu} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\beta+\mu} & 0 & 0 \\ \frac{\beta}{(\beta+\mu)(\gamma+\xi+\mu)} & \frac{1}{(\psi\varpi+\mu)(\gamma+\xi+\mu)} & 0 \\ \frac{\beta\xi}{(\psi\varpi+\mu)(\beta+\mu)(\gamma+\xi+\mu)} & \frac{\xi}{(\psi\varpi+\mu)(\gamma+\xi+\mu)} & \frac{1}{(\psi\varpi+\mu)} \end{pmatrix} - \lambda \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right| = 0 \quad (13)$$

Solving (13) above, we have

$\lambda_1 = \lambda_2 = 0$ and $\lambda_3 = \dfrac{\alpha\beta\pi}{\mu(\beta+\mu)(\gamma+\xi+\mu)}$

Hence , $R_0 = \dfrac{\alpha\beta\pi}{\mu(\beta+\mu)(\gamma+\xi+\mu)}$

Remark 2

The implication of the basic reproduction number is of two folds:

i. If $R_0 > 1$, then single corrupted person will influence more than one person to indulge in corruption activities, hence corruption will persist in the population

ii. If $R_0 < 1$, then corruption will be controlled

Local stability Analysis of the corruption – free equilibrium point

Theorem 3: *The corruption – free equilibrium point E_0, is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof

To prove local stability of corruption free equilibrium, we obtained the Jacobian matrix of the system (1) at corrupt free equilibrium E_0

Let

$$\left.\begin{aligned} f_1 &= \pi - \alpha SC - \mu S \\ f_2 &= \alpha SC + (1-\varpi)\alpha CR - (\beta+\mu)E \\ f_3 &= \beta E - (\xi+\gamma+\mu)C \\ f_4 &= \xi C - (\psi\varpi+\mu)J \\ f_5 &= \gamma C + \psi\varpi J - \big((1-\varpi)\alpha C + \mu\big)R \end{aligned}\right\} \quad (14)$$

Thus, the Jocobian matrix for the system (14) is given by the following:

$$\mathcal{J} = \begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial E} & \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial J} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial E} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial J} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial E} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial J} & \frac{\partial f_3}{\partial R} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial E} & \frac{\partial f_4}{\partial C} & \frac{\partial f_4}{\partial J} & \frac{\partial f_4}{\partial R} \\ \frac{\partial f_5}{\partial S} & \frac{\partial f_5}{\partial E} & \frac{\partial f_5}{\partial C} & \frac{\partial f_5}{\partial J} & \frac{\partial f_5}{\partial R} \end{pmatrix}$$

$$J = \begin{pmatrix} -\mu & 0 & -\alpha S & 0 & 0 \\ 0 & -(\beta+\mu) & \alpha S & 0 & 0 \\ 0 & \beta & -(\gamma+\xi+\mu) & 0 & 0 \\ 0 & 0 & \xi & -(\psi\varpi+\mu) & 0 \\ 0 & 0 & \gamma & \psi\varpi & -\mu \end{pmatrix}$$

(15)

Evaluating (7) at CFE

$$J(E_0) = \begin{pmatrix} -\mu & 0 & -\frac{\alpha\pi}{\mu} & 0 & 0 \\ 0 & -(\beta+\mu) & \frac{\alpha\pi}{\mu} & 0 & 0 \\ 0 & \beta & -(\gamma+\xi+\mu) & 0 & 0 \\ 0 & 0 & \xi & -(\psi\varpi+\mu) & 0 \\ 0 & 0 & \gamma & \psi\varpi & -\mu \end{pmatrix}$$

(16)

The characteristic equation of (16) is given as

$$(\lambda+\mu)^2(\lambda+\psi\varpi+\mu)\left(\mu(\lambda+\beta+\mu)(\lambda+\gamma+\xi)-\pi\alpha\beta\right)=0$$

(17)

Solving equation (17) for λ, we have $\lambda_1 = \lambda_2 = -\mu$, $\lambda_3 = -(\psi\varpi + \mu)$, the remaining two eigenvalues are

$$\mu\lambda^2 + \mu(\xi+\gamma+2\mu+\beta)\lambda + \mu(\beta+\mu)(\gamma+\xi+\mu) - \pi\alpha\beta = 0$$

(18)

When (18) is substituted in R_0 , we have,

$$\mu\lambda^2 + \mu(\xi+\gamma+2\mu+\beta)\lambda + \mu(\beta+\mu)(\gamma+\xi+\mu)(1-R_0) = 0$$

(19)

Rewriting equation (19) in form of
$ax^2 + bx + c = 0$

where,

$$a = \mu$$

$$b = \mu(\xi + \gamma + 2\mu + \beta)$$

$$c = \mu(\beta + \mu)(\gamma + \xi + \mu)(1 - R_0)$$

Using Routh - Hurwitz criteria, the eigenvalues of the matrix all have negative real parts, and so the system of equation (19) is locally asymptotically stable.

Endemic Equilibrium Point

The endemic equilibrium point represents persistence of corruption in the population. the endemic equilibrium point is computed in terms of the force of corruption using the model system equation (1).

From the first equation of (1), we have

$$S^* = \frac{\pi}{\alpha C^* + \mu}$$

From the third equation of (1), we got

$$E^* = \frac{(\xi+\gamma+\mu)C}{\beta}$$

From the fourth equation, we obtained

$$J^* = \frac{\xi C}{\psi\varpi + \mu}$$

From the fifth equation of (1), we have

$$R^* = \frac{\gamma(\psi\varpi + \mu)C + \psi\varpi\xi C}{(\psi\varpi + \mu)(\alpha C(\psi\varpi + \mu) + \mu)}$$

From the second equation, we have

$$C^* = C$$

Hence the endemic equilibrium point E_1 is giving by

$$E_1 = (S^*, E^*, C^*, J^*, R^*) = \left(\frac{\pi}{\alpha C+\mu}, \frac{(\xi+\gamma+\mu)C}{\beta}, C, \frac{\xi C}{\psi\varpi+\mu}, \frac{\gamma(\psi\varpi+\mu)C+\psi\varpi\xi C}{(\psi\varpi+\mu)(\alpha C(\psi\varpi+\mu)+\mu)}\right)$$

Global stability Analysis of corruption Endemic Equilibrium Point

Theorem 4: *The endemic equilibrium points E_1 of the model is globally asymptotically stable if $R_0 < 1$*

Proof

Consider the Lyapunov function about E_1

$$V\left(S^*, E^*, C^*, J^*, R^*\right) = \left(S - S^* - 1n\frac{S^*}{S}\right) + \left(E - E^* - 1n\frac{E^*}{E}\right) + \left(C - C^* - 1n\frac{C^*}{C}\right) + \left(J - J^* - 1n\frac{J^*}{J}\right) + \left(R - R^* - 1n\frac{R^*}{R}\right)$$

(20)

Differentiating (12) with respect to t, we have

$$\frac{dV}{dt} = \frac{S - S^*}{S}\frac{dS}{dt} + \frac{E - E^*}{E}\frac{dE}{dt} + \frac{C - C^*}{C}\frac{dC}{dt} + \frac{J - J^*}{J}\frac{dJ}{dt} + \frac{R - R^*}{R}\frac{dR}{dt}$$

(21)

Substituting and simplifying $\frac{dS}{dt}, \frac{dE}{dt}, \frac{dC}{dt}, \frac{dJ}{dt}, \frac{dR}{dt}$ in (21), we obtained $\frac{dV}{dt} = \chi_1 - \chi_2$

where,

$$\chi_1 = \pi + S^*\alpha C + S^*\mu + E^*\left(\beta + \mu\right) + C^*\left(\xi + \gamma + \mu\right) + J^*\left(\psi\varpi + \mu\right) + R^*\left(\left(1 - \varpi\right)\alpha C + \mu\right)$$

$$\chi_2 = \frac{S^*}{S}\pi + \mu S + \mu E + \frac{E^*}{E}\alpha SC + \frac{E^*}{E}\left(1 - \varpi\right)\alpha CR + \mu C + \frac{C^*}{C}\beta E + \mu J + \frac{J^*}{J}\xi C + \mu R + \frac{R^*}{R}\gamma C + \frac{R^*}{R}\psi\varpi J$$

$\frac{dV}{dt} \le 0$ if $\chi_1 < \chi_2$

$\frac{dV}{dt} = 0$ if and only if

$S = S^*, \quad E = E^*, \quad C = C^*, \quad J = J^*, \quad R = R^*$

Observe that, the largest invariant impact invariant set in $\{S^*, E^*, C^*, J^*, R^*\} \in \mathfrak{D}: \frac{dV}{dt} = 0$ is a singleton set. Furthermore, by Lassalle's invariant principle, it implies that E_1 is globally asymptotically stable in $\mathfrak{D}$ if $\chi_1 < \chi_2$.

Sensitivity Analysis

The sensitivity indices of the corruption reproduction number are calculated in order to determine how important each parameter is in the initiation of corruption: that is in the control of corruption, the parameters that have the greatest influence on the corruption reproduction, and the parameters that have the greatest influence on the corruption reproduction number. We adopted the local sensitivity analysis based on the normalized forward sensitivity index R_0.

Table 3: Parameter value

Parameter	Value	Source
α	0.00009	Assumed
β	3.9	Assumed
γ	1.8	Musa & Fori
ξ	20.9	Assumed
π	0.80	Alhassan
μ	0.31	Abdulrahman
ξ	20.9	Assumed
ϖ	0.035	Zerihun
ψ	1.20	Alhassan

Table 4: Sensitivity index for the parameters with respect to R_0

Parameter	Value	$\Lambda_l^{R_0}$	Sensitivity Index
α	0.3	1	$1.0000 > 0$
β	0.8	$\frac{\mu}{\beta + \mu}$	$0.0326 > 0$
γ	0.5	$-\frac{\gamma}{\xi + \gamma + \mu}$	$-0.4673 < 0$
ξ	0.3	$-\frac{\xi}{\xi + \gamma + \mu}$	$-0.3628 < 0$
π	85.5	1	$1.0000 > 0$
μ	0.0234	$-\frac{3\mu^2 + 2\mu(\beta+\gamma+\xi) + \beta(\gamma+\xi)}{\mu(\beta+\mu)(\gamma+\xi+\mu)}$	$-10.94 < 0$

Interpretation of Sensitivity Indices

The sensitivity indices of parameters are presented in Table 4. Parameters that have positive indices $(\alpha, \beta \text{ and } \pi)$ have great impact on expanding the corruption in the community when their values increase. Parameters with negative indices $(\gamma, \mu \text{ and } \xi)$ minimize the burden of corruption in the community as their values increase. Therefore, the model sensitivity analysis demonstrated that anti – corruption agencies are supposed to decrease positive index parameters and increase the negative index parameters to combat corruption in a population.

Numerical Simulation

In this section, numerical simulations of the model (1) are performed. MATLAB 2015RB software is used to validate the analytical solution in section 3.

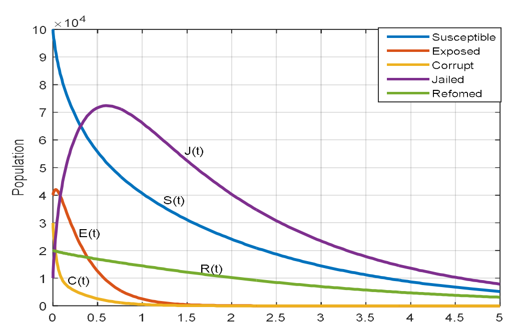

Figure 2: Time evolution of $S(t)$, $E(t)$, $C(t)$, $J(t)$, $R(t)$ with $\gamma = 1.8$ and $\varpi = 0.030$

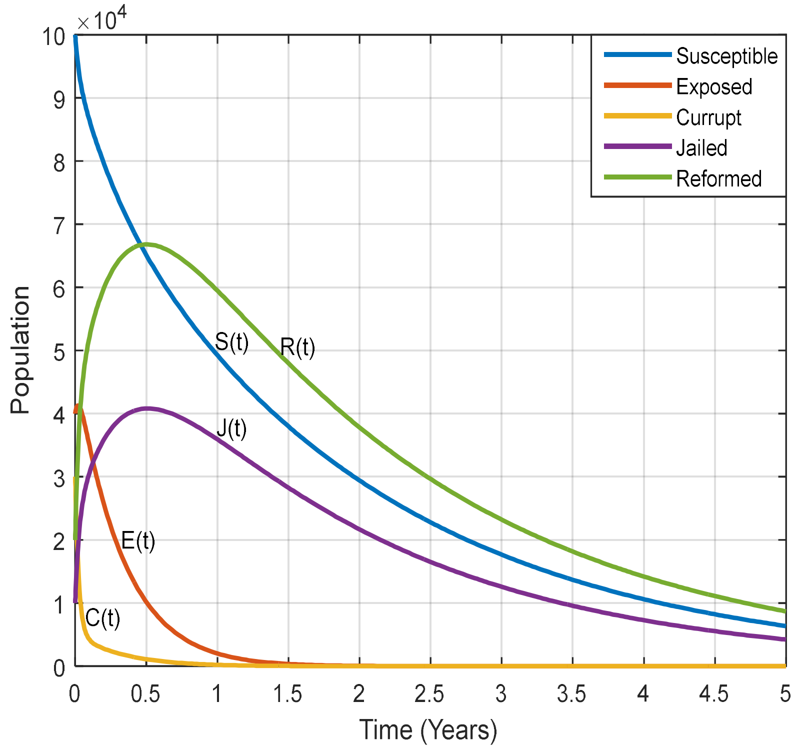

Figure 3: Time evolution of $S(t),\ E(t),\ C(t),\ J(t),\ R(t)$ with $\gamma = 35.8$ and $\varpi = 0.030$

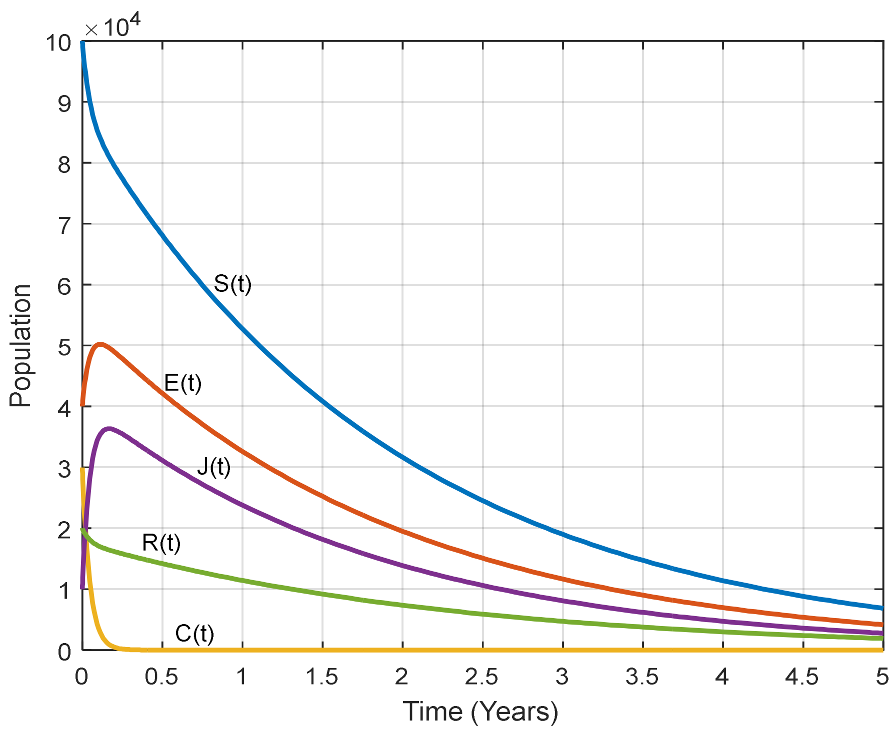

Figure 4: Time evolution of $S(t),\ E(t),\ C(t),\ J(t),\ R(t)$ with $\gamma = 0.0008$ and $\omega = 0.030$

Discussions

The simulation result indicates that an increase in the jail population directly correlates with rise in the number of exposed individuals. Furthermore, the population of the exposed individuals declines as the rate of jailing corrupt individuals ξ increases (Figure 1 & 2). The analysis also reveals that decreasing interactions between susceptible and corrupt individuals reduces the exposed group's size, while higher reformation and jailing mitigate the spread of the corruption (Figure 3). However, when a reformed individual lose immunity, the number of reformed individuals decrease leading to rise in the exposed compartment again.

Conclusions

In this paper, a mathematical model for the dynamics of corruption in population was formulated. The basic reproduction number R_0 was computed, and the stability of the equilibrium point was investigated. Using Lyapunov's function theory, the corruption - free equilibrium point is globally asymptotically stable whenever $R_0 < 1$. Using the definition of normalized forward sensitivity, the sensitivity parameters were determined. It has been shown parameters that have positive indices have great impact on expanding the corruption in the community when their values increase, and parameters with negative indices minimize the burden of corruption in the community as their values increase.

Recommendations

A loss of immunity among reformed individuals leads to an increase in the exposed population. To mitigate this, a two – integrated strategy is recommended:

i. Significantly reduce contact rates between susceptible and corrupt populations.

ii. Increase reform and jail rates to curb corruption. Furthermore, consider all recommendations suggested by Binuyo and Akinsola, 2020.

Sources of Data

Data for this research was obtained from secondary sources and have been dully cited, whereas others has been assumed.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this study.

Acknowledgements

The Tertiary Education Trust Fund (TETFUND) Federal Republic of Nigeria supported the research work. It was financially supported by the IBR TETFund Research Project (Batch -04 2024).

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